Brief Summary of NOWS Theoretical Backgrounds

⦿ Power spectral density matrix of longitudinal wind velocity fluctuation

▣ Power spectral density function of longitudinal wind velocity fluctuations – two-sided form

- Kaimal et al. (1972); Simiu (1974); Simiu and Scanlan (1996),

$$S_{rr}(z,\omega) = \frac{1}{2}\cdot\frac{200}{2\pi}\cdot u_*^2\cdot\frac{z}{U(z)}\cdot\frac{1}{\left[1+50\cdot\frac{\omega z}{2\pi U(z)}\right]^{5/3}}$$

where, z = height, ω = circular frequency (rad/s); u_* = friction velocity; $U(z)$ = mean wind speed at height z

▣ Coherence function (two-dimensional)

- Davenport (1967); Simiu and Scanlan (1996)

$$f_{rs}(\omega) = \exp\left[-\frac{\omega}{2\pi}\frac{C_z\Delta z}{\frac{1}{2}\left[U(z_r)+U(z_s)\right]}\right]\cdot\exp\left[-\frac{\omega}{2\pi}\frac{C_x\Delta x}{\frac{1}{2}\left[U(x_r)+U(x_s)\right]}\right]$$

where, x, z = horizontal and vertical directions, respectively; Δz = $|z_r - z_s|$; Δx = $|x_r - x_s|$; C_z, C_x = a constant, generally taken 10 and 16 for structural design viewpoint, respectively.

▣ Cross-spectral density function

- Co-spectrum (quadratic term of wind is ignored)

$$S_{rs}(\omega) = \sqrt{S_{rr}(\omega)\cdot S_{ss}(\omega)}\,\exp(-f_{rs}(\omega))$$

▣ Power spectral density matrix $S(\omega)$: two-dimensional, n-variate

$$S(\omega) = \begin{bmatrix} S_{11}(\omega) & S_{12}(\omega) & \cdots & S_{1n}(\omega) \\ S_{21}(\omega) & S_{22}(\omega) & \cdots & S_{2n}(\omega) \\ \vdots & \vdots & \cdots & \vdots \\ S_{n1}(\omega) & S_{n2}(\omega) & \cdots & S_{nn}(\omega) \end{bmatrix}$$

⦿ Simulation schemes of wind velocity fluctuations

1. Discrete frequency function with FFT

– Wittig and Sinha (1975)

Discrete time series can be simulated using the following model:

$$y_p(n\Delta t) = \frac{1}{N}\sum_{k=0}^{N} Y_p(k\Delta f)\exp\left(j\frac{2\pi kn}{N}\right)$$

$$Y_p(k\Delta f) = \sum_{i=0}^{p} H_{pi}(k\Delta f)\varepsilon_{ik}\sqrt{2 f_c N}$$

where,

$H_{pi}(k\Delta f)$: a lower triangular matrix by Cholesky decomposition of one-sided power spectral density function $S(f)$; $\varepsilon_{ik} = \xi_{ik} + j\eta_{ik}$ = complex Gaussian random number with zero mean and 0.5 variance; $\Delta t = \frac{1}{2 f_c}$; f_c = Nyquist frequency

2. Schur decomposition approach with AR (autoregressive)

– Di Paola (1998); Di Paola and Gullo (2001)

The n-variate stochastic vector process $V(t)$ can be decomposed into a summation of n-variate fully coherent normal vectors $Y_j(t)$ independent of each other:

$$V(t) = \sum_{j=1}^{n} Y_j(t)$$

Let $\psi(\omega)$ be the eigenmatrix of $S(\omega)$ whose columns are the eigenvectors (real and orthogonal), then, following relationship holds:

$$\Psi^T(\omega)S(\omega)\Psi(\omega) = \Lambda(\omega)$$
$$\Psi^T(\omega)\Psi(\omega) = I$$

Vectors $Y_j(t)$ can be described as:

$$Y_j(t) = \int_{-\infty}^{\infty} S(\omega)e^{i\omega t}dB_j(\omega) = \int_{-\infty}^{\infty} \psi_j(\omega)\sqrt{\Lambda(\omega)}e^{i\omega t}dB_j(\omega)$$

Let define the frequency domain [ω_0, ω_c], where ω_0 and ω_c are lower and upper cut-off frequencies, and subdivided the domain into M parts $\omega_0 \equiv \Omega_0, \Omega_1, \ldots, \Omega_m \equiv \omega_c$

With third-order polynomial approximation of eigenvectors $\psi_j^{(s)}(\omega)$,

$$\psi_j^{(s)}(\omega) = N_j^{(s)} l(\omega), \quad \Omega_{s-1} \le \omega \le \Omega_s$$

where, $l(\omega)$=[1 ω ω^2 ω^3].

Accordingly, vectors $Y_j(t)$ can be expressed as:

$$Y_j(t) = \sum_{s=1}^{M} N_j^{(s)} \int_{\Omega_{s-1}}^{\Omega_s} l(\omega)\sqrt{\Lambda(\omega)}e^{i\omega t}dB_j^{(s)}(\omega) = \sum_{s=1}^{M} N_j^{(s)} U_j^{(s)}(t)$$

where, $U_j^{(s)}(t) = \sum_{s=1}^{M} \int_{\Omega_{s-1}}^{\Omega_s} l(\omega)\sqrt{\Lambda(\omega)}e^{i\omega t}dB_j^{(s)}(\omega)$

Using the standard generation via AR(autoregressive) model:

$$U_{j,r}^{(s)}(t_k) = \sum_{u=1}^{p} a_{j,u}^{(s)} U_{j,r}^{(s)}(t_{k-u}) + \sigma_{j,1}^{(s)} W_j^{(s)}(t_k)$$

where r = 1,…,4; $a_{j,u}^{(s)}$ = parameters of the AR model; $\sigma_{j,r}^{(s)}$ = variances of the input; $W_j^{(s)}$ = normal random variables with zero mean and unit variance; p = AR model order

AR parameters can be evaluated by using Yule-walker scheme, that is, autocorrelation method.

It is worth noting that in this study, M = 1 and p = 4 are used to generate wind velocity fluctuations.

3. Ergodic spectral representation method

– Deodatis (1996); Ding et al. (2006)

Power spectral density matrix $S(\omega)$ can be decomposed into the following product:

$$S(\omega) = H(\omega)H^{T*}(\omega)$$

where, $H(\omega)$ is a lower triangular matrix by Cholesky decomposition of $S(\omega)$

Stochastic process $V_j(t)$ can be described by using following trigonometric series:

$$V_j(t) = 2\sum_{m=1}^{j}\sum_{l=1}^{N}\left|H_{jm}(\omega)\right|\sqrt{\Delta\omega}\cos[\omega_{ml}t - \theta_{jm}(\omega_{ml}) + \Phi_{ml}]$$

To take advantage of the FFT technique, above equation can be rewritten as follows:

$$V_j(p\Delta t) = \mathrm{Re}\left\{\sum_{m=1}^{j} h_{jm}(p\Delta t)\exp\left[i\left(\frac{m\Delta\omega}{n}\right)(p\Delta t)\right]\right\}]$$

where, N = number of n-variate simulation; j = 1, 2,…, N; p = 0, 1, … , $n \times (M - 1)$; $M = 2N$;

$$h_{jm}(p\Delta t) = \begin{cases} g_{jm}(p\Delta t) & for\ \ p = M, M+1, \ldots, 2M-1 \\ g_{jm}[(p-M)\Delta t] & for\ \ p = M, M+1, \ldots, 2M-1 \\ \vdots & \vdots \\ g_{jm}[(p-nM)\Delta t] & for\ \ p = nM, n(M+1), \ldots, nM-1 \end{cases}$$

where,

$$g_{jm}(p\Delta t) = \sum_{l=0}^{M-1} B_{jml}\exp\left[ilp\frac{2\pi}{M}\right] \qquad p = 0,1,\ldots,M-1$$

$$B_{jml} = 2\left|H_{jm}(\omega_{ml})\right|\sqrt{\Delta\omega}\exp\left[-i\theta_{jm}(\omega_{ml})\right]\exp(i\Phi_{ml})$$

$$\omega_{ml} = (l-1)\Delta\omega + \frac{m}{n}\Delta\omega \qquad m = 1,2,\ldots,n; \quad l = 1,2,\ldots,N$$

$$\theta_{jm}(\omega_{ml}) = \tan^{-1}\left\{\frac{\mathrm{Im}(H(\omega_{ml}))}{\mathrm{Re}(H(\omega_{ml}))}\right\}$$

Φ_{ml} = independent random phase angles distributed uniformly over the interval [0, 2π]

Note : $g_{jm}(p\Delta t)$ can be obtained from inverse FFT of B_{jml}

4. Conventional spectral representation method

– Shinozuka and Deodatis (1991)

Power spectral density matrix $S(\omega)$ can be decomposed into the following product:

$$S(\omega) = H(\omega)H^{T*}(\omega)$$

where, $H(\omega)$ is a lower triangular matrix by Cholesky decomposition of $S(\omega)$.

Stochastic process $V(t)$ can be described by using following trigonometric series:

$$V(t) = \sqrt{2}\sum_{n=0}^{N-1} A_n \cos\left(\omega_n t + \phi_n\right)$$

To take advantage of the FFT technique, above equation can be rewritten as follows:

$$V(p\Delta t) = \mathrm{Re}\left\{\sum_{n=0}^{M-1} B_n \exp\left[i\left(n\Delta\omega\right)(p\Delta t)\right]\right\}$$

where, $A_n = \sqrt{\left(2\cdot\Delta\omega\right)}\left|H(\omega)\right|$, N = number of n-variate simulation; p = 0, 1, … , M - 1;

M = 2N; $B_n = 2\left|H(\omega)\right|\sqrt{\Delta\omega}\cdot\exp\left(i\phi_n\right)$; $\Delta\omega$ = ω_u/N; ω_u = upper cut-off frequency [rad/sec];

ϕ_n = independent random phase angles distributed uniformly over the interval [0, 2π]

⦿ Selected references

Davenport, A. G. (1967). “The dependence of wind load upon meteorological parameters.” *Proc. International Research Seminar on Wind Effects on Building and Structures*, University of Toronto Press, Toronto, 19-82.

Deodatis, G (1996). “Simulation of ergodic multivariate stochastic processes.” *Journal of Engineering Mechanics*, 122(8), 778-787.

Di Paola, M. (1998). “Digital simulation of wind field velocity.” *Journal of Wind Engineering and Industrial Aerodynamics*, 74–76, 91−109.

Di Paola, M. and Gullo, I. (2001). “Digital generation of multivariate wind field processes.” *Probabilistic Engineering Mechanics*, 16, 1-10.

Ding, Q., Zhu, L., and Xiang, H. (2006), “Simulation of stationary Gaussian stochastic wind velocity field.” *Wind and Structure*, 9(3), 231-243.

Kaimal, J. C., Wyngaard, J. C., Izumi, Y. and Cote, O. R. (1972). “Spectral characteristics of surface-layer turbulence.” *Journal of Royal Meteorological Society*, 98, 563-589.

Li, Y. and Kareem, A. (1993). “Simulation of multivariate random processes : Hybrid DFT and digital filtering approach.” *Journal of Engineering Mechanics*, 119(5), 1078-1098

Simiu, E. (1974). “Wind spectra and dynamic alongwind response.” *J. Struct. Div.*, ASCE, 100(9), 1897-1910.

Shinozuka, M and Deodatis, G. (1991). “Simulation of stochastic processes by spectral representation.” *Applied Mechanics Reviews*, 44(4), 191–204.

Simiu, E, and Scanlan, R. H. (1996). *Wind effects on structures*, Third edition, John Wiley & Sons, Inc.

Wittig, L. E. and Sinha, A. K. (1975). “Simulation of multicorrelated random processes using the FFT algorithm.” *The Journal of the Acoustical Society of America*, 58(3), 630−633.